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REFLECTION AND REFRACTION OF SOUND WAVES AT A WATER-SOLID INTERFACE

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REFLECTION AND REFRACTION OF SOUND WAVES AT A WATER-SOLID INTERFACE

by

D.K. Mak* and C.J. Schramm**

ABSTRACT

When a sound beam strikes an interface between materials of different acoustic velocity or impedance properties, longitudinal and shear waves are reflected and refracted at the interface. A computer program has been written to compute the amplitude and energy of the waves reflected and refracted at the water/metal interface and the metal/water interface of a metal block immersed in water. The time of flight and the path travelled by the longitudinal and shear waves in the metal are also computed.

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LABORATOIRES DE RECHERCHE EN MÉTALLURGIE PHYSIQUE
RAPPORT ERL/PMRL 84-2(TR)

RÉFLEXION ET RÉFRACTION D'ONDES SONORES À UNE INTERFACE EAU-SOLIDE

par

D.K. Mak* et C.J. Schramm**

RÉSUMÉ

Lorsqu'un faisceau sonore se heurte à une interface entre des matières de vitesse acoustique et d'impédance différentes, les ondes longitudinales et ondes de cisaillement sont réfléchies et réfractées à l'interface. Un programme machine a été rédigé pour calculer l'amplitude et l'énergie des vagues réféchies et réfractées à l'interface eau/métal et à l'interface métal/eau d'un bloc de métal immergé dans l'eau. Ce temps de vol et la distance parcourue par les ondes longitudinales et les ondes de cisaillament dans le métal sont aussi calculés.

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INTRODUCTION

When a solid is inspected for defects by a transmitter transducer it is best if the path and energy of the reflected wave is known so that the position of the receiver transducer can be optimized to receive the maximum echo reflected.

Mode conversion occurs at interfaces and generates both longitudinal and shear waves, the amplitudes of which depend on the angle of incidence of the incident wave. Both of these modes of vibration can be detected. It is thus important to find the amplitude and energy of the sound waves reflected and refracted at the interfaces at different angles of incidence.

Three cases have been considered here:-

1. Longitudinal (progressive) waves incident in water against a solid.
2. Longitudinal waves incident in solid against water.
3. Shear waves incident in solid against water.

All the above cases occur when a sound beam traverses through a metal block immersed in water. A computer program has been written to compute the interaction of the sound waves at the water/metal and metal/water interfaces. Sound waves are considered to be plane waves. Attenuation of sound energy in metal is ignored for simplicity. The program can be easily modified to take account of attenuation. It is assumed that the metal surfaces are parallel and unbounded. However, the program can be extended to accomodate a metal block of finite size with non-parallel surfaces.

REFLECTION AND REFRACTION AT WATER-METAL INTERFACES

The study of seismic waves reflected and refracted at a discontinuity has been studied by various geophysicists. Aspects of wave propagation that are of interest in seismology are discussed in books by Ewing et al (1) and Brekhovshikh (2). The theory in seismology can be applied to ultrasonics in nondestructive testing. Ergin (3) has published a paper on seismic waves reflected and refracted at a rock-water boundary. The same principle can be applied to the reflection and refraction of acoustic waves at a liquid-solid interface.

In the following analysis, the approach and notation of Achenbach (4) is followed to derive the equations suitable for reflection and refraction of ultrasonic waves at the water-metal interface.

JOINED HALF-SPACES

A plane harmonic displacement wave has a displacement vector given by:

$$\bar{u} = A\,\bar{d}e^{i\eta} \tag{1}$$

where
$$\eta = k\,(\bar{x}.\bar{p} - ct) \tag{2}$$

Equation (1) describes a plane wave propagating with phase velocity c in a direction defined by the unit propagation vector $\bar{p}$. The unit vector defining the direction of motion is $\bar{d}$. There are two types of plane harmonic waves.

(a) Longitudinal waves for which $\bar{d} = \pm \bar{p}$ and $c = c_L$.
(b) Transverse waves for which $\bar{d}.\bar{p} = 0$ and $c = c_T$.

c_L and c_T are the longitudinal and transverse velocities respectively.

By substituting the components of (1) into Hooke's law, the components of the stress tensor are obtained as:

$$\tau_{lm} = [\lambda\,\delta_{lm}\,(d_j p_j + \mu\,(d_l p_m + d_m p_l)]\,ik\,Ae^{i\eta} \tag{3}$$

where the summation convention is invoked. τ_{lm} is the stress applied in the m direction to a unit area of a plane whose normal lies in the l direction. λ and μ are the Lamé elastic constants.

Considering in-plane motion (x_1 x_2 plane), the system of incident, reflected and refracted waves must satisfy four conditions of continuity on the displacements and the stresses at the interface of the two half-spaces.

$$[u_j] = 0 \tag{4}$$
$$[\tau_{2j}] = 0 \tag{5}$$

where $j = 1,2$.

Each incident wave will generally cause two reflected and two refracted waves. The unit propagation vectors of the system of incident, reflected and refracted waves are shown in Fig. 1.

The incident as well as the reflected and refracted wave are represented by:

$$\bar{u}^{(n)} = A_n \, \bar{d}^{(n)} \exp(i\eta_n) \tag{6}$$

where different values of the index n serve to label the various types of waves that occur, and where

$$\eta_n = k_n (x_1 p_1^{(n)} + x_2 p_2^{(n)} - c_n t) \tag{7}$$

The index n is assigned the value n = 0 for the incident wave, i.e.,

$$\bar{p}^{(o)} = \sin\theta_o \, \bar{i}_1 + \cos\theta_o \, \bar{i}_2 \tag{8}$$

If the incident wave is a longitudinal wave we have:

$$\bar{d}^{(o)} = \bar{p}^{(o)}, \qquad c_o = c_L \tag{9}$$

For an incident transverse wave we have:

$$\bar{d}^{(o)} \cdot \bar{p}^{(o)} = 0, \qquad c_o = c_T \tag{10}$$

We consider only "vertically" polarized transverse (shear) waves (SV-waves). An incident SV-wave is defined by:

$$\bar{d}^{(o)} = \bar{i}_3 \wedge \bar{p}^{(o)}, \qquad c_o = c_T \tag{11}$$

The reflected and refracted waves may be both longitudinal and transverse waves. The reflected longitudinal waves and the reflected transverse

waves are labeled by n = 1 and n = 2, respectively. The refracted longitudinal and transverse waves are labeled by n = 3 and n = 4, respectively. Thus we have:

Reflected longitudinal wave:

$$\bar{p}^{(1)} = \sin\theta_1\,\bar{I}_1 - \cos\theta_1\,\bar{I}_2 \quad , \quad \bar{d}^{(1)} = \bar{p}^{(1)} \tag{12}$$

Reflected transverse wave:

$$\bar{p}^{(2)} = \sin\theta_2\,\bar{I}_1 - \cos\theta_2\,\bar{I}_2 \quad , \quad \bar{d}^{(2)} = \bar{I}_3 \wedge \bar{p}^{(2)} \tag{13}$$

Refracted longitudinal wave:

$$\bar{p}^{(3)} = \sin\theta_3\,\bar{I}_1 + \cos\theta_3\,\bar{I}_2 \quad , \quad \bar{d}^{(3)} = \bar{p}^{(3)} \tag{14}$$

Refracted transverse wave:

$$\bar{p}^{(4)} = \sin\theta_4\,\bar{I}_1 + \cos\theta_4\,\bar{I}_2 \quad , \quad \bar{d}^{(4)} = \bar{I}_3 \wedge \bar{p}^{(4)} \tag{15}$$

(A) LONGITUDINAL WAVE INCIDENT IN WATER AGAINST THE METAL

As water slides freely along the surface of the metal, tangential displacement is not continuous, i.e. $[u_1] \neq 0$. And as water does not support shear waves, $\mu = 0$ and $A_2 = 0$.

Three conditions of continuity have to be satisfied at $x_2 = 0$:

normal displacement $$u_2^{(o)} + u_2^{(1)} = u_2^{(3)} + u_2^{(4)} \tag{16}$$

tangential stress $$\tau_{21}^{(o)} + \tau_{21}^{(1)} = \tau_{21}^{(3)} + \tau_{21}^{(4)} \tag{17}$$

normal stress $$\tau_{22}^{(o)} + \tau_{22}^{(1)} = \tau_{22}^{(3)} + \tau_{22}^{(4)} \tag{18}$$

Equations (16) to (18) must be valid for all values of x_1 and t, which implies:

$$\bar{n}_o = \bar{n}_1 = \bar{n}_3 = \bar{n}_4 \tag{19}$$

where $\bar{n}_n = \bar{n}_n\ (x_2 = 0)$.

Using (7), (19) implies:

$$k_o \sin \theta_o = k_1 \sin \theta_1 = k_3 \sin \theta_3 = k_4 \sin \theta_4 \tag{20}$$

$$k_o c_L = k_1 c_L = k_3 c_L^B = k_4 c_T^B \tag{21}$$

For a given θ_o amd k_o, the other angles and wavenumbers can thus be obtained from (20) and (21).

Equations (16)-(18) form a system of three equations for the amplitudes A_1, A_3 and A_4. In matrix notation the system can be written as:

$$\begin{bmatrix} \cos\theta_o & \cos\theta_3 & \sin\theta_4 \\ 0 & \mu^B \frac{c_L}{c_L^B} \sin 2\theta_3 & -\mu^B \frac{c_L}{c_T^B} \cos 2\theta_4 \\ -\lambda & \mu^B \frac{c_L}{c_L^B} \left(\frac{c_L^B}{c_T^B}\right)^2 \cos 2\theta_4 & \mu^B \frac{c_L}{c_T^B} \sin 2\theta_4 \end{bmatrix} \begin{bmatrix} A_1 \\ A_3 \\ A_4 \end{bmatrix} = A_o \begin{bmatrix} \cos\theta_o \\ 0 \\ \lambda \end{bmatrix} \tag{22}$$

The material constants of the metal are labeled by the superscript B.

As
$$c_L^2 = \frac{\lambda}{\rho} \tag{23}$$

$$c_L^{B2} = \frac{\lambda^B + 2\mu^B}{\rho^B} \quad \text{and} \quad c_T^{B2} = \frac{\mu^B}{\rho^B} \tag{24}$$

Equation (22) can be written in an alternate form which was the one used in the computer program.

$$\begin{bmatrix} \cos\theta_o & \cos\theta_3 & \sin\theta_4 \\ 0 & \sin 2\theta_3 & \frac{-c_L^B}{c_T^B}\cos 2\theta_4 \\ -1 & \frac{\rho^B}{\rho}\frac{c_L^B}{c_L}\cos 2\theta_4 & \frac{\rho^B}{\rho}\frac{c_T^B}{c_L}\sin 2\theta_4 \end{bmatrix} \begin{bmatrix} \frac{A_1}{A_o} \\ \frac{A_3}{A_o} \\ \frac{A_4}{A_o} \end{bmatrix} = \begin{bmatrix} \cos\theta_o \\ 0 \\ 1 \end{bmatrix} \qquad (25)$$

It should be noted from the law of conservation of energy that:

$$\frac{A_1^2}{A_o^2} + \frac{A_3^2}{A_o^2}\left(\frac{\rho^B \sin 2\theta_3}{\rho \sin 2\theta_o}\right) + \frac{A_4^2}{A_o^2}\left(\frac{\rho^B \sin 2\theta_3}{\rho \sin 2\theta_o}\right) = 1 \qquad (26)$$

The square root of these energy ratios are plotted versus the angle of incidence in Figs. 2-4.

When $\theta_3 = 90°$, $\theta_o = \theta_c$ and total internal reflection occurs. For $\theta_o > \theta_c$, $\sin\theta_3 > 1$ and $\cos\theta_3$ becomes imaginary. The matrix equation thus becomes complex.

(B) LONGITUDINAL WAVE INCIDENT IN METAL AGAINST WATER

As there is no shear wave refracted into the water, $A_4 = 0$.

The equations, in matrix notation, can be written as:

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_2 & \cos\theta_3 \\ \sin 2\theta_1 & \kappa^B \cos 2\theta_2 & 0 \\ -(\kappa^B)^2 \cos 2\theta_2 & \kappa^B \sin 2\theta_2 & \frac{\lambda}{B_\mu}\frac{c_L^B}{c_L} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = A_o \begin{bmatrix} \cos\theta_o \\ \sin 2\theta_o \\ (\kappa^B)^2 \cos\theta_2 \end{bmatrix} \qquad (27)$$

Here $\kappa^B = \frac{c_L^B}{c_T^B}$

and A_o is the amplitude of the longitudinal wave incident in metal.

Equation (27) can be written in a modified form which was of the format used in the program.

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_2 & \cos\theta_3 \\ \sin 2\theta_1 & \frac{c_L^B}{c_T^B}\cos 2\theta_2 & 0 \\ -\cos 2\theta_2 & \frac{c_T^B}{c_L^B}\sin 2\theta_2 & \frac{c_L}{c_L^B}\frac{\rho}{\rho^B} \end{bmatrix} \begin{bmatrix} \frac{A_1}{A_o} \\ \frac{A_2}{A_o} \\ \frac{A_3}{A_o} \end{bmatrix} = \begin{bmatrix} \cos\theta_o \\ \sin 2\theta_o \\ \cos 2\theta_2 \end{bmatrix} \tag{28}$$

For a given angle of incidence θ_o, the following equations must be used to obtain the other angles:

$$k_o \sin\theta_o = k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 \tag{29}$$

$$k_o c_L^B = k_1 c_L^B = k_2 c_T^B = k_3 c_L \tag{30}$$

It should be noted that:

$$\frac{A_1^2}{A_o^2} + \frac{A_2^2}{A_o^2}\frac{\sin 2\theta_2}{\sin 2\theta_o} + \frac{A_3^2\,\rho\sin 2\theta_3}{A_o^2\,\rho^B \sin 2\theta_o} = 1 \tag{31}$$

The square root of these energy ratios are plotted versus the angle of incidence in Figs. 5-7.

(C) TRANSVERSE (SV) WAVE INCIDENT IN METAL AGAINST WATER

As there is no shear wave refracted in the water, $A_4 = 0$.

The equations, in matrix notation, can be written as:

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_2 & \cos\theta_3 \\ (\kappa^B)^{-1}\sin 2\theta_1 & \cos 2\theta_2 & 0 \\ -\kappa^B \cos 2\theta_2 & \sin 2\theta_2 & \frac{\lambda}{\mu^B}\frac{c_T^B}{c_L} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_2 \end{bmatrix} = A_o \begin{bmatrix} \sin\theta_o \\ -\cos 2\theta_o \\ \sin 2\theta_o \end{bmatrix} \quad (32)$$

Here A_o is the amplitude of the shear wave incident in the metal.

Equation (32) can be rewritten in a different form which was of the format used in the program:

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_2 & \cos\theta_3 \\ \frac{c_T^B}{c_L^B}\sin 2\theta_1 & \cos 2\theta_2 & 0 \\ \frac{-c_L^B}{c_T^B}\cos 2\theta_2 & \sin 2\theta_2 & \frac{c_L \rho}{c_T^B \rho^B} \end{bmatrix} \begin{bmatrix} \frac{A_1}{A_o} \\ \frac{A_2}{A_o} \\ \frac{A_3}{A_o} \end{bmatrix} = \begin{bmatrix} \sin\theta_o \\ -\cos 2\theta_o \\ \sin 2\theta_o \end{bmatrix} \quad (33)$$

For a given angle of incidence θ_o, the following equations are needed to obtain the other angles:

$$k_o \sin\theta_o = k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 \quad (34)$$

$$k_o c_T^B = k_1 c_L^P = k_2 c_T^B = k_3 c_L \quad (35)$$

It should again be noted that:

$$\frac{A_2^2}{A_o^2} + \frac{A_1^2}{A_o^2}\,\frac{\sin 2\theta_1}{\sin 2\theta_o} + \frac{A_3^2}{A_o^2}\,\frac{\rho}{\rho_B}\,\frac{\sin 2\theta_3}{\sin 2\theta_o} = 1 \qquad (36)$$

Square roots of these energy ratios are plotted versus the angle of incidence in Figs. 8-10.

Figures 2-10 compare quite well quantitatively to those of Ergin's (3). At points where discrepancies occur, we estimate that our computation is more accurate than Ergin's as we used a computer and computation was carried out at intervals of 1/16 of a degree of the angle of incidence. Ergin used a slide rule for his computation.

RAY TRACING THROUGH A METAL BLOCK

The principles and equations were applied in a program designed to trace the path of an acoustic wave, incident in water at an angle to be supplied by the user, as it travels through a block of a specified metal (Fig. 11). The sides of the block are assumed to be parallel and unbounded; the thickness, the density of the metal and the velocities of the longitudinal and transverse waves in the metal are supplied as input parameters. The results are tabulated for every possible wave-boundary combination: a longitudinal wave at the water-metal interface, a longitudinal wave and a transverse wave at the metal-water interface. In each case, the angles of reflection and refraction, the amplitude ratio and the square root of the energy ratio with respect to the original incident wave are given for each resultant reflected and refracted wave. The times of flight of the longitudinal and shear waves through the metal are also given.

An example of the output is listed below:

REFLECTION AND REFRACTION FOR SOUND WAVES AT SOLID - WATER BOUNDARIES

SYMBOLS USED

P : Longitudinal waves

SV: Transverse waves

CONSTANTS USED IN COMPUTATIONS

Water

Velocity of P waves: 1480.00 m/s

Density: 1000.00 kg/m^3

Structural Steel

Velocity of P waves: 5940.00 m/s

Velocity of SV waves: 3250.00 m/s

Density: 7850.00 kg/m^3

Thickness: 10.00 cm

Amplitude of Incident P Wave: 1.00

THE WATER - SOLID BOUNDARY

Angle of Incidence of P Wave: 10.00 degrees

	Incident P wave in water	Reflected P wave in water	Refracted P wave in solid	Refracted SV wave in solid
Angle (degrees)	10.00	10.00	44.18	22.42
Amplitude ratio	1.00000	0.93728	0.06108	0.04711
Square root of energy ratio	1.00000	0.93728	0.29257	0.18949

THE SOLID - WATER BOUNDARY

P Wave In Solid

Angle of incidence: 44.18 degrees
Position: (9.72, -10.00) (cm)
Time of travel: .2348E-04 s.

	Incident P wave in solid	Reflected P wave in solid	Reflected SV wave in solid	Refracted P wave in water
Angle (degrees)	44.18	44.18	22.42	10.00
Amplitude ratio	0.06108	0.02228	0.06429	0.08560
Square root of energy ratio	0. 7257	0.10673	0.25861	0.08560

SV Wave In Solid

Angle of incidence: 22.42 degrees
Position: (4.12, -10.00) (cm)
Time of travel: .3328E-04 s.

	Incident SV wave in solid	Reflected P wave in solid	Reflected SV wave in solid	Refracted P wave in water
Angle (degrees)	22.42	44.18	22.42	10.00
Amplitude ratio	0.04711	0.03497	0.02014	0.03591
Square root of energy ratio	0.18949	0.16749	0.08101	0.03591

CONCLUSIONS

The above computer program can be adapted to obtain the energy ratio of sound waves reflected and refracted from any surfaces. The surfaces have to be described by using analytical geometry. One practical application will be inspecting defects by using the tandem method of ultrasonic inspection.

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3. K. Ergin, Bulletin of the Seismological Society of America, 42, 349; 1952.

4. J.D. Achenbach, Wave Propagation in Elastic Solids; North-Holland; 1973.

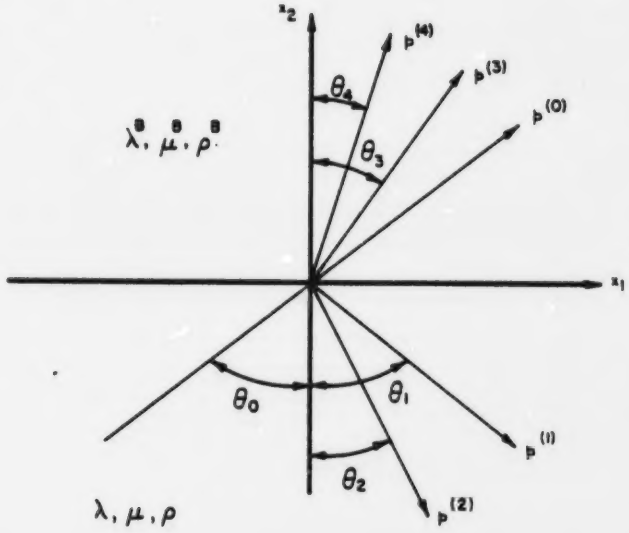
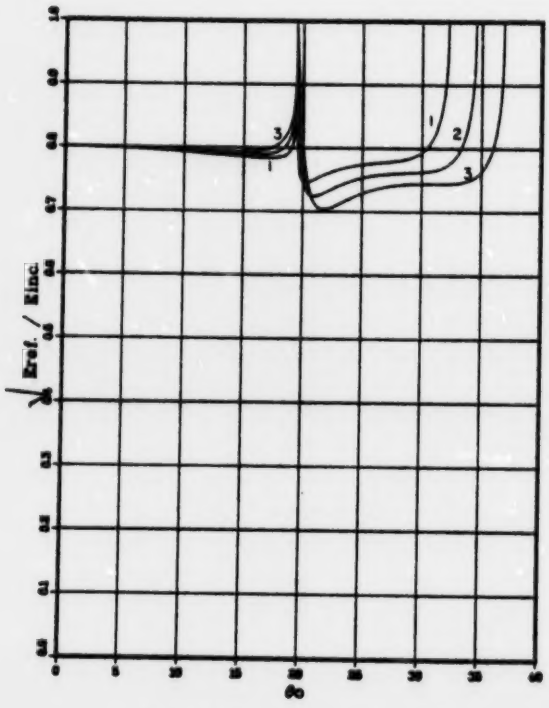
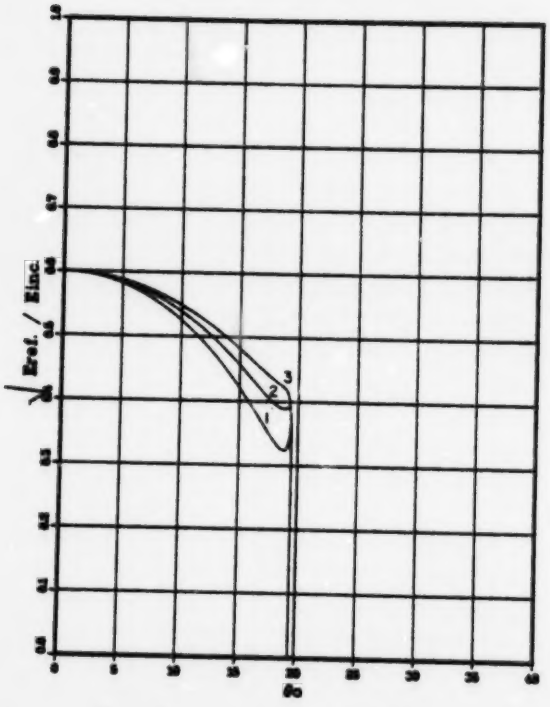


Fig. 1. Incident, reflected and refracted waves.. (neg 9108)



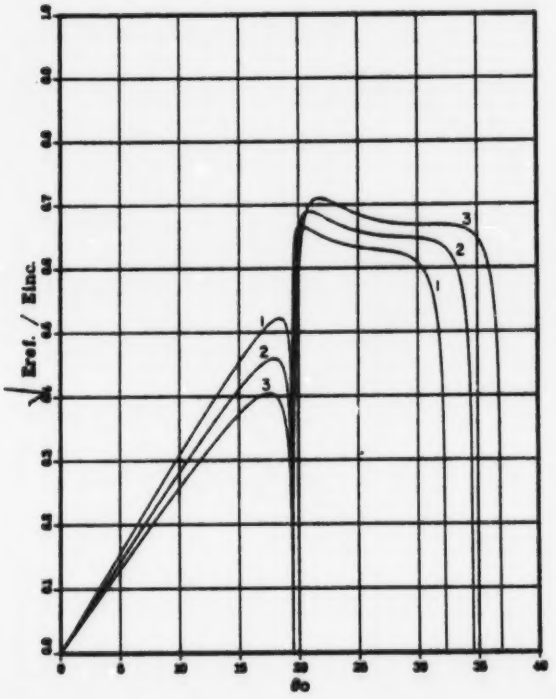
Curve	c_L^B/c_T^B	c_L^B/c_L	ρ^B/ρ
1	1.6	3.0	3.0
2	1.7	3.0	3.0
3	1.8	3.0	3.0

Fig. 2 - P wave incident in water against solid. Square root of the energy ratio of the reflected P wave. (neg. 9115)



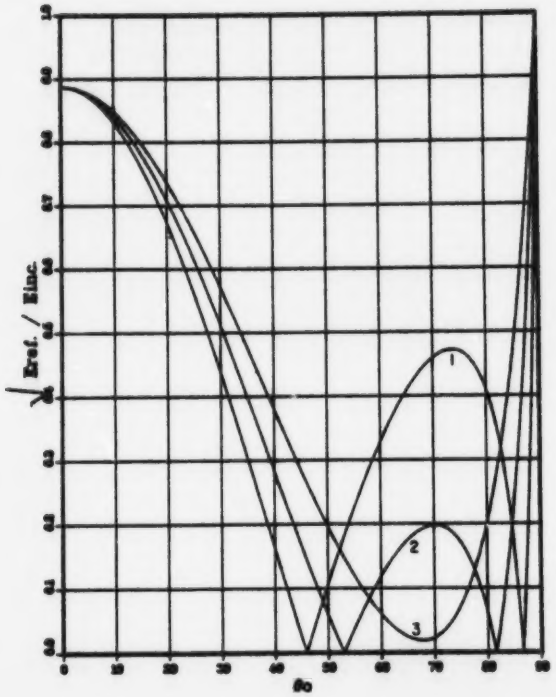
Curve	c_L^B/c_T^B	c_L^B/c_L	ρ^B/ρ
1	1.6	3.0	3.0
2	1.7	3.0	3.0
3	1.8	3.0	3.0

Fig. 3 - P wave incident in water against solid. Square root of the energy ratio of the refracted P wave. (neg. 9117)



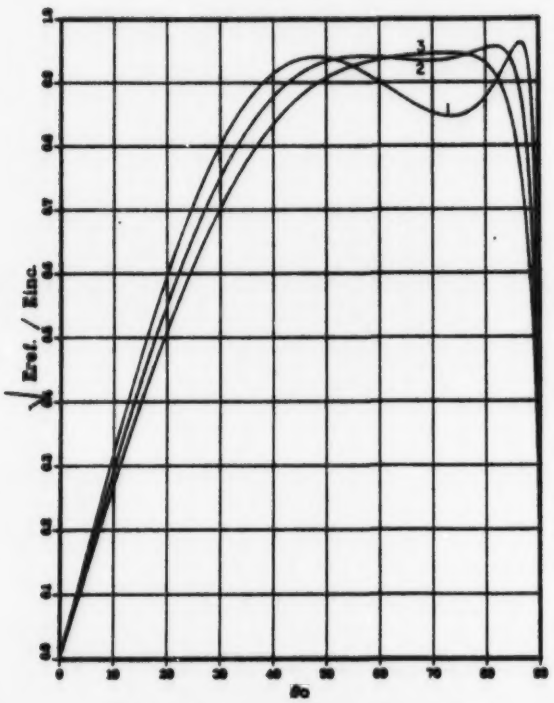
Curve	c_L^B/c_T^B	c_L^B/c_L	ρ^B/ρ
1	1.6	3.0	3.0
2	1.7	3.0	3.0
3	1.8	3.0	3.0

Fig. 4 - P wave incident in water against solid. Square root of the energy ratio of the refracted SV wave. (neg. 9109)



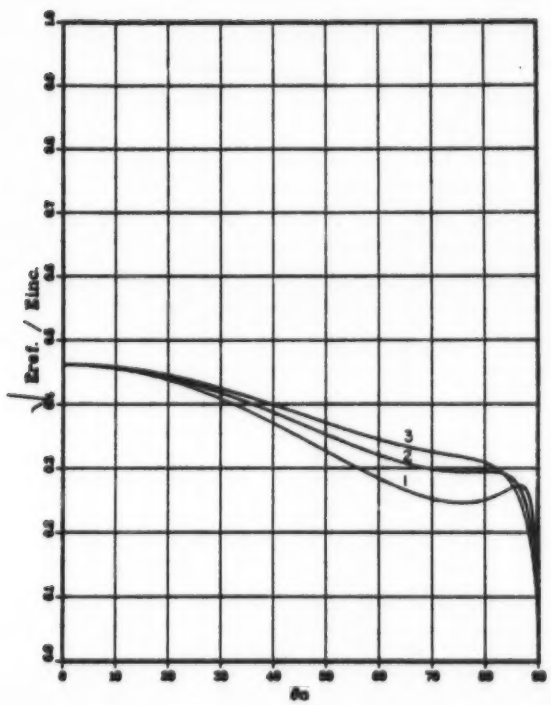
Curve	c_L^B/c_T^B	c_L/c_L^B	ρ/ρ^B
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.8	0.2	0.3

Fig. 5 - P wave incident in solid against water. Square root of the energy ratio of the reflected P wave. (neg. 9110)



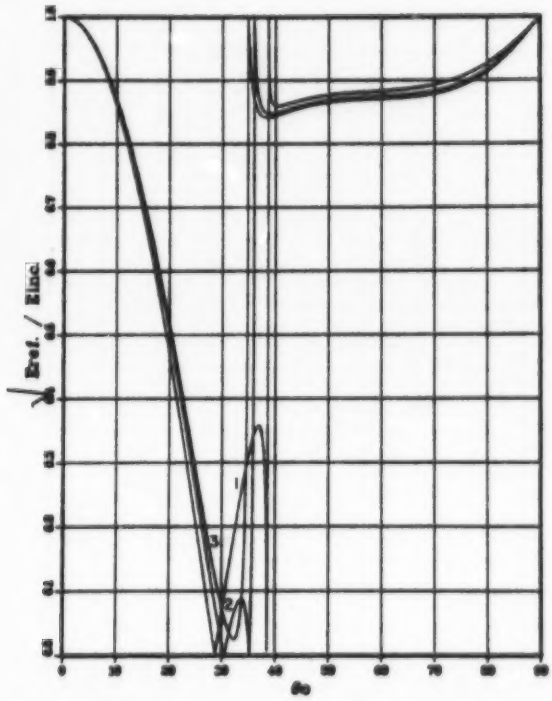
Curve	c_L^B/c_T^B	c_L/c_L^B	ρ/ρ^B
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.8	0.2	0.3

Fig. 6 - P wave incident in solid against water. Square root of the energy ratio of the reflected SV wave. (neg. 9111)



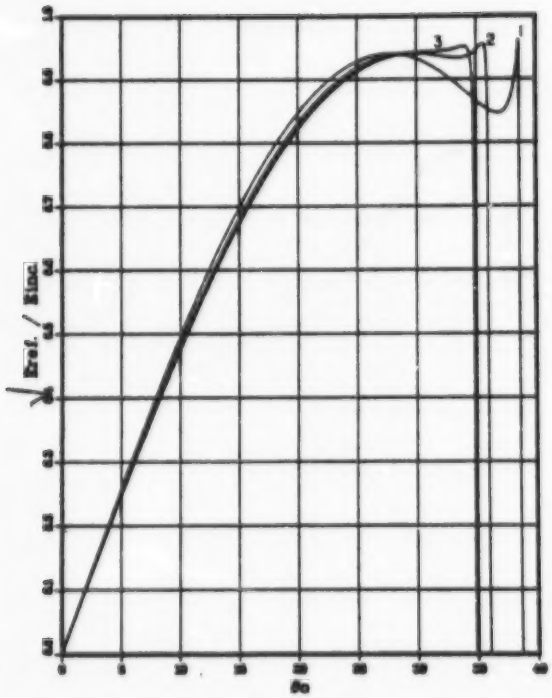
Curve	c_L^B/c_T^B	c_L/c_L^B	ρ/ρ^B
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.8	0.2	0.3

Fig. 7 - P wave incident in solid against water. Square root of the energy ratio of the refracted P wave. (neg. 9112)



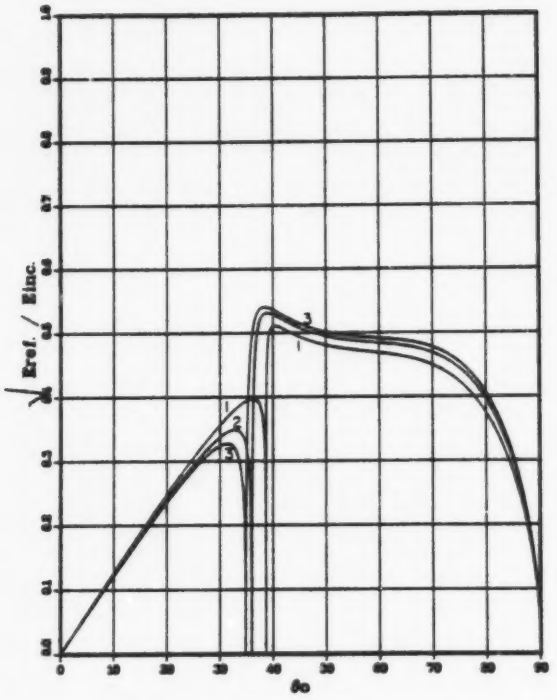
Curve	c_L^B/c_T^B	c_L/c_L^B	ρ/ρ^B
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.75	0.2	0.3

Fig. 8 - SV wave incident in solid against water. Square root of the energy ratio of the reflected SV wave. (neg. 9114)



Curve	c_L^B/c_T^B	c_L/c_L^B	ρ/ρ^B
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.75	0.2	0.3

Fig. 9 - SV wave incident in solid against water. Square root of the energy ratio of the reflected P wave. (neg. 9118)



Curve	c_L^B/c_T^B	c_L/c_L^B	ρ/ρ^B
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.75	0.2	0.3

Fig. 10 - SV wave incident in solid against water. Square root of the energy ratio of the refracted P wave. (neg. 9113)

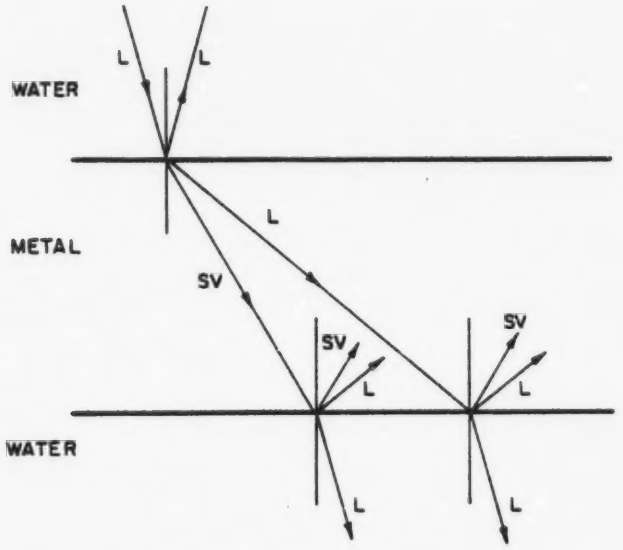


Fig. 11. Longitudinal and shear waves reflected and refracted at the water-metal and metal-water interfaces of a metal block. (neg 9116)

POOR

QUALITY

ORIGINAL